**Electrodynamics in Metal-Insulator**

And now we’ll plug our model for the charges and currents into Maxwell’s equations.

**Equations presuming constant local susceptibilities**

We’ll start with time-independent, constant susceptibilities (not to say the fields are consant, time-independent entities). We’ll write out the bound/induced charges and currents in terms of the susceptibilities and such. And then we’ll plug these back into ME’s.



**Susceptibilities**

So first we’ll write out the charge density. Like before, the bound charge is basically the divergence of the polarization. And we presume a linear susceptibility between polarization and total field.



and linear P-E relation as before:



As long as we’re in the low frequency range ω << ωp = plasma frequency (~1015Hz), ω << ω0 = resonant frequency of bound electrons, then the static approximation to the polarization should be okay. For the bound current we have the same as before.



Along with the linear relations:



where as before, H is given by:



But this isn’t the only contribution to the bound current. If we have changing fields, then we have changing charge densities too. And this would also result in a current, **j**ʹb, via:



It’s actually impertinent to equate these two. All we can say is that they’re equal up to the curl of some vector field, since the divergence of the curl is zero. But looks like we can get away with it. So we’ll say:



And then of course we have the induced current from Ohm’s law:



We’ll presume σ to be constant – not time-dependent. This assumes that our fields are low frequency on the scale of ωτ = 1/τ­­ ~ 1014Hz. But that’s not all. From Ohm’s law, and the continuity equation (recall from the Metal Physical Model file), if we have a spatially changing electric field, then we could have an associated charge density. Consider:



But we’ll typically presume ρind to be negligible, because any induced ρ will typically decay to zero over a very short period. We can see why. If we presume that we have only induced charges in our metal, for simplicity, then we can say ρ = ρind, and so ∇·E = ρ/ε0 = ρind/ε0:



and this implies ρind ~ ρ0e-(σ/ε0)t. Now typical values of σ/ε0 are (ρ is resistivity here):



and so this would tend to make any accumulated charge density decay to zero within a time frame of

10-19 seconds. So it would seem that unless the field is oscillating with a frequency greater than ~ 1019 Hz, we shouldn’t need to worry about the metallic ρind, nor its concomittant induced current either (but really, probably don’t need to worry about frequencies greater than this either because the charge fluctuations will get smaller as ω increases). And this coheres with what we found in the Metal σ(w) file – that neglecting diffusion (setting D → 0) resulted in an imperceptible change in behavior, at least as far as absorption was concerned. So altogether,



**Maxwell’s Equations**

Now we want to fill these into ME’s



For the top one we have:



The middle two are unchanged,



and the last one is:



So all together,



If we specialize to a linear medium so that **D** = ε**E** and **H** = **B**/μ where ε = ε0(1+χe) and μ = μ0(1+χm), and further, to one where the susceptibilities are position-independent so that we can pull them outside the spatial derivatives, we can say:



Boundary conditions should be as usual:



**Equations presuming nonconstant local susceptibilities**

First we’ll write out the bound charges and currents in terms of the susceptibilities and such. And then we’ll plug these back into ME’s.

**Susceptibilities**

So first we’ll write out the charge density. Like before, the bound charge is basically the divergence of the polarization. And we presume a linear susceptibility between polarization and total field. Note I’m leaving some arguments of variables off, but we presume all E and B fields depend on space and time (and same with polarization, magnetizations, etc.).



and linear P-E relation as before:



We will find, as usual, that these relationships are best expressed in Fourier space (see previous files). So taking the time-dependent Fourier transform, this comes to:



(don’t forget that χe is material-dependent and so can technically change with position, and **E** can be as well, though its position dependence is being notationally suppressed) For the bound current, we have as usual:



where we now update the M-H relationship as we did the P-E one:



Again the temporal Fourier transform is helpful. So,



and on the M-H equation,



Along with the equation,



and its Fourier transform



We can solve for H in terms of B:



And then last we have Ohm’s law. The conductivity relation becomes when we have an alternating/time-dependent **E**.



The Fourier transform of both sides says,



And if we were interested in the induced charge response, we could say,



A Fourier transform of both sides will turn this to:



But again, we will usually neglect this guy. And so can say, altogether:



which is, in Fourier space,



**Maxwell’s Equations**

Now we want to fill these into ME’s



To get tractable equations, we’ll have to take the temporal Fourier transforms,



So plugging our stuff into the top one:



where we recall the definition of **D**(ω) = ε0**E**(ω)+ **P**(ω) [just the Fourier transform of the equation **D**(t) = ε0**E**(t) + **P**(t), again suppressing position arguments]. The two middle equations are simply:



and the last one is:



So altogether, we have:



If we postulate a linear medium so that **D**(ω) = ε0**E**(ω)+ **P**(ω) = ε0**E**(ω)+ ε0χe(ω)**E**(ω) = ε0(1+χe(ω))**E**(ω) = ε(ω)**E**(ω), and **H**(ω) = **B**(ω)/μ(ω), and further that the susceptibilities are not spatially dependent, so that they can be pulled outside derivatives, we would have:



where we define an effective dielectric constant that incorporates the conductivity



Again, position arguments are being suppressed. Boundary conditions should be as usual:



**Absorption Spectrum**

Let’s calculate the power absorbed by the system in the presence of a field **E**(**r**,t), which I’ll take to be Re[**E**(**r**,ω)e-iωt]. We should have for the time-averaged power:



Now the induced current is given by – leaving out the probably negligible magnetic contribution:



So we can say,



(can add the ε0 ‘cause we’re taking the real part ultimately)



We’ll recall the quantity in brackets was defined to be the effective dielectric susceptibility. So now we have:



We can identify the quantity intrinsic to the material, which contributes to the power, as the absorptivity, A(ω):



It’s basically the time-averaged power absorbed per unit volume per unit field modulus. If we presume ε(ω) is constant, or just real, then certainly this reduces to:

